

Generalized second law of thermodynamics in modified FRW cosmology with corrected entropy-area relation

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April 22, 2011

Abstract

Using the corrected entropy-area relation motivated by the loop quantum gravity, we investigate the validity of the generalized second law of thermodynamics in the framework of modified FRW cosmology. We consider a non-flat universe filled with an interacting viscous dark energy with dark matter and radiation. The boundary of the universe is assumed to be the dynamical apparent horizon. We find out that the generalized second law is always satisfied throughout the history of the universe for any spatial curvature regardless of the dark energy model.

PACS numbers: 95.36.+x, 04.60.Pp

Key words: Dark energy; Loop quantum gravity

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1 Introduction

It is quite possible that the gravitational field equations for the spacetime metric have a predisposition to thermodynamic behavior. This profound connection between gravity and thermodynamics was first addressed by Jacobson who disclosed that the hyperbolic second order partial differential Einstein equation can be derived from the relation between the horizon area and entropy, together with the Clausius relation $\delta Q = T\delta S$ [1]. The investigations on the deep connection between gravity and thermodynamics have been generalized to the cosmological context where it has been shown that the differential form of the Friedmann equation in the FRW universe can be written in the form of the first law of thermodynamics on the apparent horizon [2, 3, 4, 5, 6, 7, 8, 9]. See [10] for a recent review on thermodynamical aspects of gravity.

If thermodynamical interpretation of gravity near the apparent horizon is a generic feature, one needs to verify whether the results may hold not only for more general spacetimes but also for the other principles of thermodynamics, especially the generalized second law (GSL) of thermodynamics as a global accepted principle in the universe. The GSL of thermodynamics is an important principle in governing the nature. Recently the GSL in the accelerating universe enveloped by the apparent horizon has been investigated in [11, 12, 13]. It was argued [12] that in contrast to the case of the apparent horizon, the GSL of thermodynamics breakdown if one consider the universe to be enveloped by the event horizon with the usual definitions of entropy and temperature. This study reveals that in an accelerating universe with spatial curvature, the apparent horizon is a physical boundary from the thermodynamical point of view. Using the general expression of temperature at apparent horizon of FRW universe, it has been shown that the GSL holds in Einstein, Gauss-Bonnet and more general lovelock gravity [14]. The GSL of thermodynamics has also been studied in the framework of braneworld [15]. Other studies on the GSL of thermodynamics have been carried out in [16, 17, 18].

It is interesting to note that Friedmann equations, in Einstein's gravity, can be derived by using Clausius relation to the apparent horizon of FRW universe, in which entropy is assumed to be proportional to its horizon area, $S = A/4$ [4]. However, this definition for entropy can be modified from the inclusion of quantum effects, motivated from the loop quantum gravity (LQG). The quantum corrections provided to the entropy-area relationship leads to the curvature correction in the Einstein-Hilbert action and vice versa [19]. The corrected entropy takes the form [20]

$$S_A = \frac{A}{4} - \alpha \ln \frac{A}{4} + \beta \frac{4}{A}, \quad (1)$$

where α and β are positive dimensionless constants of order unity. The exact values of these constants are not yet determined and still an open issue in loop quantum cosmology. These corrections arise in the black hole entropy in LQG due to thermal equilibrium fluctuations and quantum fluctuations [21]. Taking into account the quantum corrections in entropy expression, it has been shown that the Friedmann equations will be modified as well [22, 23, 24]. The cosmological implications of the modified Friedmann equations have also been studied in different setup [25, 26].

In the present work we would like to examine whether the corrected entropy-area relation, from LQG, together with the matter field entropy inside the apparent horizon will

satisfy the GSL of thermodynamics. To be more general we will consider an interacting viscous dark energy (DE) with dark matter (DM). In addition, we will also include the contribution from the radiation. In an isotropic and homogeneous FRW universe, the dissipative effects arise due to the presence of bulk viscosity in cosmic fluids. The theory of bulk viscosity was initially investigated by Eckart [27] and later on pursued by Landau and Lifshitz [28]. DE with bulk viscosity has a peculiar property to cause accelerated expansion of phantom type in the late evolution of the universe [29]. It can also alleviate several cosmological puzzles like age problem, coincidence problem and phantom crossing.

The structure of this paper is as follows. In the next section we consider the modified Friedmann equation for a universe filled with viscous DE, DM and radiation. In section 3, we examine the validity of the GSL for the modified Friedmann equation corresponding to the corrected entropy-area relation. We finish with conclusions in the last section.

2 Interacting viscous DE, DM and radiation in modified FRW cosmology

In the framework of Friedmann-Robertson-Walker (FRW) metric,

$$ds^2 = -dt^2 + a^2(t) \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right), \quad (2)$$

for the non-flat FRW universe containing the DE, DM and radiation, the modified first Friedmann equation corresponding to the corrected entropy-area relation (1) is given by [22]

$$H^2 + \frac{k}{a^2} - \frac{\alpha}{2\pi} \left(H^2 + \frac{k}{a^2} \right)^2 - \frac{\beta}{3\pi^2} \left(H^2 + \frac{k}{a^2} \right)^3 = \frac{8\pi}{3} (\rho_D + \rho_m + \rho_r), \quad (3)$$

where we take $G = 1$ and $k = 0, 1, -1$ represent a flat, closed and open universe, respectively. Also ρ_D , ρ_m and ρ_r are the energy density of DE, DM and radiation, respectively.

From Eq. (3), we can write

$$1 + \Omega_k = \Omega_D + \Omega_m + \Omega_r + \Omega_\alpha + \Omega_\beta, \quad (4)$$

where we have used the following definitions

$$\Omega_k = \frac{k}{a^2 H^2}, \quad \Omega_D = \frac{8\pi \rho_D}{3H^2}, \quad \Omega_m = \frac{8\pi \rho_m}{3H^2}, \quad \Omega_r = \frac{8\pi \rho_r}{3H^2}, \quad (5)$$

$$\Omega_\alpha = \frac{\alpha}{2\pi} H^2 (1 + \Omega_k)^2, \quad \Omega_\beta = \frac{\beta}{3\pi^2} H^4 (1 + \Omega_k)^3. \quad (6)$$

Here we would like to consider the viscous DE model. Dissipative processes are thought to be present in any realistic theory of the evolution of the universe. The observations also indicate that the universe media is not a perfect fluid and the viscosity is concerned in the evolution of the universe (see [30] and references therein). In the framework of FRW metric, the shear viscosity has no contribution in the energy-momentum tensor, and the

bulk viscosity behaves like an effective pressure [31]. The total energy density satisfies a conservation law

$$\dot{\rho} + 3H(\rho + p) = 0, \quad (7)$$

where

$$\rho = \rho_D + \rho_m + \rho_r, \quad (8)$$

$$p = \tilde{p}_D + p_r, \quad (9)$$

and

$$\tilde{p}_D = p_D - 3H\xi, \quad (10)$$

is the effective pressure of the DE and ξ is the bulk viscosity coefficient [30, 31]. Note that the DM is pressureless, i.e. $p_m = 0$. Here like [32], if we assume $\xi = \varepsilon \rho_D H^{-1}$, where ε is a constant parameter, then the total pressure can be written as

$$p = (\omega_D - 3\varepsilon)\rho_D + \frac{1}{3}\rho_r, \quad (11)$$

where $\omega_D = p_D/\rho_D$ is the equation of state (EoS) parameter of the viscous DE.

We consider the case where the viscous DE, DM and radiation interact with each other. In some recent studies, the scenario in which the DE interacts with DM and radiation has been introduced to resolve the cosmic triple coincidence problem [33]. Interaction causes the DE, DM and radiation do not conserve separately and they must rather enter the energy balances

$$\dot{\rho}_D + 3H\rho_D(1 + \omega_D) = 9H^2\xi - Q', \quad (12)$$

$$\dot{\rho}_m + 3H\rho_m = Q, \quad (13)$$

$$\dot{\rho}_r + 4H\rho_r = Q' - Q, \quad (14)$$

where Q and Q' stand for the interaction terms.

Taking time derivative in both sides of Eq. (3), and using Eqs. (4), (5), (6), (12), (13) and (14), one can get the EoS parameter of interacting viscous DE as

$$\omega_D = -\frac{1}{3\Omega_D} \left[2 \left(\frac{\dot{H}}{H^2} - \Omega_k \right) \left(1 - \frac{2\Omega_\alpha + 3\Omega_\beta}{1 + \Omega_k} \right) + 3\Omega_m + 4\Omega_r \right] + 3\varepsilon - 1. \quad (15)$$

The deceleration parameter is given by

$$q = - \left(1 + \frac{\dot{H}}{H^2} \right). \quad (16)$$

Replacing the term \dot{H}/H^2 from (15) into (16) yields

$$q = \frac{(1 + \Omega_k)}{2(1 + \Omega_k - 2\Omega_\alpha - 3\Omega_\beta)} \left[3\Omega_D(1 + \omega_D - 3\varepsilon) + 3\Omega_m + 4\Omega_r \right] - (1 + \Omega_k). \quad (17)$$

Using Eq. (4) one can rewrite (17) as

$$q = \frac{(1 + \Omega_k)}{2(1 + \Omega_k - 2\Omega_\alpha - 3\Omega_\beta)} \left[1 + \Omega_k + \Omega_\alpha + 3\Omega_\beta + 3\Omega_D(\omega_D - 3\varepsilon) + \Omega_r \right]. \quad (18)$$

3 GSL with corrected entropy-area relation

Here, we study the validity of the GSL of thermodynamics for the entropy-corrected Friedman equation. According to the GSL, entropy of the viscous DE, DM and radiation inside the horizon plus the entropy associated with the apparent horizon do not decrease with time. Define $\tilde{r}(t) = a(t)r$, the metric (2) can be rewritten as $ds^2 = h_{ij}dx^i dx^j + \tilde{r}^2 d\Omega^2$, where $x^i = (t, r)$, $h_{ij} = \text{diag}(-1, a^2/(1 - kr^2))$, $i, j = 0, 1$. By definition

$$f := h^{ij}\partial_i \tilde{r} \partial_j \tilde{r} = 1 - \left(H^2 + \frac{k}{a^2} \right) \tilde{r}^2, \quad (19)$$

when $f = 0$ then the location of the apparent horizon in the FRW universe is obtained as [34]

$$\tilde{r}_A = H^{-1}(1 + \Omega_k)^{-1/2}. \quad (20)$$

The surface gravity for the FRW universe is obtained as [4]

$$\kappa = -\frac{1}{\tilde{r}_A} \left(1 - \frac{\dot{\tilde{r}}_A}{2H\tilde{r}_A} \right). \quad (21)$$

The associated Hawking temperature on the apparent horizon is defined as

$$T_A = \frac{|\kappa|}{2\pi} = \frac{1}{2\pi\tilde{r}_A} \left(1 - \frac{\dot{\tilde{r}}_A}{2H\tilde{r}_A} \right), \quad (22)$$

where $\frac{\dot{\tilde{r}}_A}{2H\tilde{r}_A} < 1$ ensure that the temperature is positive.

Taking time derivative in both sides of Eq. (20), and using Eqs. (3), (4), (5), (6), (7), (8) and (11), one can get

$$\dot{\tilde{r}}_A = \frac{(1 + \Omega_k)^{-1/2}}{2(1 + \Omega_k - 2\Omega_\alpha - 3\Omega_\beta)} \left[3\Omega_D(1 + \omega_D - 3\varepsilon) + 3\Omega_m + 4\Omega_r \right]. \quad (23)$$

Using Eq. (17) one can rewrite (23) as

$$\dot{\tilde{r}}_A = \frac{1 + \Omega_k + q}{(1 + \Omega_k)^{3/2}}. \quad (24)$$

From Eqs. (1), (22) and using $A = 4\pi\tilde{r}_A^2$, the evolution of the apparent horizon entropy is obtained as

$$T_A \dot{S}_A = 4\pi H \tilde{r}_A^3 (\rho + p) - 2\pi \tilde{r}_A^2 \dot{\tilde{r}}_A (\rho + p). \quad (25)$$

The entropy of the universe including the viscous DE, DM and radiation inside the dynamical apparent horizon can be related to its energy and pressure in the horizon by Gibbs equation [16]

$$TdS = d(\rho V) + p dV = Vd\rho + (\rho + p)dV, \quad (26)$$

where $V = 4\pi\tilde{r}_A^3/3$ is the volume of the universe enclosed by the dynamical apparent horizon \tilde{r}_A . Following [15, 17], we limit ourselves to the assumption that the thermal

system bounded by the dynamical apparent horizon remain in equilibrium so that the temperature of the system must be uniform and the same as the temperature of its boundary. This requires that the temperature T of the universe enclosed by the dynamical apparent horizon should be in equilibrium with the Hawking temperature T_A associated with the dynamical apparent horizon, so we have $T = T_A$. Therefore from Eq. (26) one can obtain

$$T_A \dot{S} = 4\pi \tilde{r}_A^2 \dot{\tilde{r}}_A (\rho + p) - 4\pi H \tilde{r}_A^3 (\rho + p), \quad (27)$$

where $S = S_D + S_m + S_r$ is the entropy in the universe containing the viscous DE, DM and radiation. Finally, adding Eqs. (25) and (27), the GSL due to the different contributions of the viscous DE, DM, radiation and dynamical apparent horizon can be obtained as

$$T_A \dot{S}_{\text{tot}} = 2\pi \tilde{r}_A^2 \dot{\tilde{r}}_A (\rho + p), \quad (28)$$

where $S_{\text{tot}} = S + S_A$ is the total entropy.

From Eqs. (8), (11) and using (5) one can obtain

$$\rho + p = \frac{H^2}{8\pi} [3\Omega_D(1 + \omega_D - 3\varepsilon) + 3\Omega_m + 4\Omega_r]. \quad (29)$$

Using Eq. (17) one can rewrite (29) as

$$\rho + p = \frac{H^2}{4\pi} \left(\frac{1 + \Omega_k + q}{1 + \Omega_k} \right) (1 + \Omega_k - 2\Omega_\alpha - 3\Omega_\beta). \quad (30)$$

Substituting Eqs. (20), (24) and (30) into (28) yields the GSL as

$$T_A \dot{S}_{\text{tot}} = \frac{(1 + \Omega_k - 2\Omega_\alpha - 3\Omega_\beta)}{2(1 + \Omega_k)^{7/2}} (1 + \Omega_k + q)^2, \quad (31)$$

which can be rewritten by the help of Eq. (17) as

$$T_A \dot{S}_{\text{tot}} = \frac{1}{8} \frac{[3\Omega_D(1 + \omega_D - 3\varepsilon) + 3\Omega_m + 4\Omega_r]^2}{(1 + \Omega_k)^{3/2} (1 + \Omega_k - 2\Omega_\alpha - 3\Omega_\beta)}. \quad (32)$$

According to Eq. (32), the validity of GSL, i.e. $T_A \dot{S}_{\text{tot}} > 0$, depends on the sign of expression $(1 + \Omega_k - 2\Omega_\alpha - 3\Omega_\beta)$ appeared in the denominator. From Eqs. (6) and (20) we have

$$1 + \Omega_k - 2\Omega_\alpha - 3\Omega_\beta = (1 + \Omega_k) \left(1 - \frac{\alpha}{\pi \tilde{r}_A^2} - \frac{\beta}{\pi^2 \tilde{r}_A^4} \right). \quad (33)$$

Following [24], it has been shown that

$$\left(1 - \frac{\alpha}{\pi \tilde{r}_A^2} - \frac{\beta}{\pi^2 \tilde{r}_A^4} \right) > 0, \quad (34)$$

because $\tilde{r}_A \gg 1$ while $\alpha \sim O(1)$ and $\beta \sim O(1)$, thus $\frac{\alpha}{\pi \tilde{r}_A^2} \ll 1$ and $\frac{\beta}{\pi^2 \tilde{r}_A^4} \ll 1$.

Finally from Eqs. (32)-(34) one can conclude that $T_A \dot{S}_{\text{tot}} > 0$. Therefore the GSL for the modified FRW universe containing the interacting viscous DE with DM and radiation enclosed by the dynamical apparent horizon is always satisfied throughout the history of the universe for any spatial curvature and it is independent of the EoS parameter of interacting viscous DE model.

4 Conclusions

Here, we investigated the validity of the GSL of thermodynamics for an entropy-corrected FRW universe filled with viscous DE and DM as well as radiation. Following the method developed in [15], we examined time evolution of the total entropy including the entropy associated with the apparent horizon plus the entropy of viscous DE, DM and radiation inside the apparent horizon. We found out that the GSL of thermodynamics in a modified FRW universe with entropy correction terms is fulfilled throughout the history of the universe. The satisfaction of the GSL of thermodynamics for the modified FRW cosmology further supports the thermodynamical interpretation of gravity and provides more confidence on the profound connection between gravity and thermodynamics.

Acknowledgements. The works of K. Karami and A. Sheykhi have been supported financially by Research Institute for Astronomy and Astrophysics of Maragha, Iran.

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